

Supporting Analysis for a Proposed Methodology to Quantify MSS/FS Interference

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1 Overview

The following sections provide the supporting analysis for a methodology developed to quantify the impact of MSS space-to-Earth interference on FS radio-relay ground stations. Specifically, Section 1.1 documents the derivation of key probability equations for the cases of link fade only and link fade plus MSS interference. Section 1.2 includes the methodology developed to transform PDF data relating to the received noise power level into PDF data relating to the level by which the received noise power level exceeds the receiver noise floor. This transformed PDF data is needed as input into the interference analysis. Finally, Section 1.3 provides several alternative methods for specifying MSS/FS interference criteria.

1.1 *Derivation of Key Probability Equations*

This section provides the derivations for key equations used in the MSS system interference analysis. We begin in section 1.1.1 by addressing the case where there is the possibility of random signal fades on the link but no signal interference from MSS space stations. An equation is derived that generates the probability that a given fade value, in dB, exceeds the link margin. In section 1.1.2 we include the presence of MSS signal interference. Specifically, an equation is derived that generates the probability that the sum of the link fade and the interference from an MSS system exceeds the link margin.

1.1.1 Case 1: No MSS Interference (i.e., Random Link Signal Fading Only)

This case has been analyzed previously and documented in ITU-R P.530. The reason for its inclusion here is to introduce several concepts and variable definitions that will be needed in Section 1.1.2 when we include signal interference from a mobile satellite system.

In general, we want to calculate the probability that the received signal-to-noise ratio (SNR) is less than the required SNR:

Equation 1 $P\{SNR_{rcv} < SNR_{req}\}$

SNR_{req} represents the signal-to-noise ratio required at the receiver on a given link to ensure a desired level of performance. SNR_{rcv} is a random process that represents the actual received signal-to-noise ratio at the receiver, taking into consideration random multi-path fading, and is equal to:

Equation 2
$$SNR_{rcv} = \frac{SNR_{link}}{A} = \frac{C/N}{A} = \frac{P_t G_t G_r / NL_p}{A}$$

where:

- C/N = Carrier to Noise power ratio (unitless)
- N = Noise Power (watts)
- P_t = Transmit Power (watts)
- G_t = Transmit Gain (unitless)
- G_r = Receive Gain (unitless)
- L_p = Path Loss (unitless)
- A = An r.v. that represents the link fade depth (expressed as a unitless factor)

and SNR_{link} represents the ideal signal-to-noise ratio at the receiver in benign conditions (i.e., without fading taken into account). Substituting the results of Equation 2 into Equation 1 we obtain:

Equation 3
$$P\{SNR_{rcv} < SNR_{req}\} = P\left\{A > \frac{SNR_{link}}{SNR_{req}}\right\}$$

If we define the following:

- A_{dB} = An r.v. that represents the link fade depth (expressed in dB)
- $SNR_{rcv,dB}$ = SNR_{rcv} in dB
- $SNR_{req,dB}$ = SNR_{req} in dB
- $SNR_{link,dB}$ = SNR_{link} in dB

we can write:

$$P\{SNR_{rcv,dB} < SNR_{req,dB}\} = P\{A_{dB} > M_{dB}\}$$

Equation 4

$$\equiv p_w(M_{dB})$$

where M_{dB} is the link margin, in dB, and is simply the difference between the link SNR and the required SNR, both in dB (i.e., M_{dB} is equal to $SNR_{link,dB} - SNR_{req,dB}$).

Equation 4 above states that the probability that the received SNR is less than the required SNR, both measured in dB, is equal to the probability that the fade experienced on the link is greater than the link margin measured in dB, M_{dB} . We define this latter probability as the function, p_w , and note that it is dependent upon M_{dB} . The importance of this observation will become apparent in the following section when MSS interference is considered. An expression is derived in ITU-R P.530 that can be used to quantify Equation 4 above. That is, ITU-R P.530 provides a methodology to calculate the probability that a given fade depth, A , is exceeded in the average worst month where A is given in dB.

1.1.2 Case 2: Link Fade Plus MSS Interference

We take that same approach as applied above. That is, we determine the probability that the received SNR is less than the required SNR. However, in this case, we take into consideration both the random fade depth and MSS interference. Thus, Equation 2 is re-written as follows:

$$\text{Equation 5} \quad SNR_{rcv} = \frac{SNR_{link}}{A} = \frac{C/N + I}{A} = \frac{P_t G_t G_r}{P_t G_t G_r (N + I) L_p}$$

where I is an r.v. that represents the MSS interference power in Watts and the other variables were defined previously. Before proceeding, it will help to clearly define the key r.v.'s that will be used in the proceeding derivation:

I = An r.v. that represents the level of interference power, in Watts, arriving from the mobile satellite system (MSS) at the input to the fixed service (FS) receiver

I_{dBW} = An r.v. that represents the level of interference power, in dBW, arriving from the mobile satellite system (MSS) at the input to the fixed service (FS) receiver

I'' = An r.v. that represents the amount that the MSS interference power exceeds the noise floor, N, at the input to the FS receiver (expressed as a dimensionless power ratio)

I''_{dB} = An r.v. that represents the amount that the MSS interference power exceeds the noise floor, N, at the input to the FS receiver (expressed in dB)

We now define I'' in terms of I and N as follows:

Equation 6
$$I'' = \frac{N + I}{N}$$

Substituting the results of Equation 5 and Equation 6 into Equation 1 results in the following:

$$\begin{aligned}
 P \{ SNR_{rcv} < SNR_{req} \} &= P \left\{ \frac{P_t G_t G_r}{L_p A (N + I)} < SNR_{req} \right\} \\
 &= P \left\{ \frac{P_t G_t G_r}{L_p A N I''} < SNR_{req} \right\} \\
 &= P \left\{ \frac{L_p A N I''}{P_t G_t G_r} > \frac{1}{SNR_{req}} \right\} \\
 &= P \left\{ A I'' > \frac{(P_t G_t G_r / N L_p)}{SNR_{req}} \right\} \\
 &= P \left\{ A I'' > \frac{SNR_{link}}{SNR_{req}} \right\}
 \end{aligned}$$

Equation 7

If we now apply the definitions of A_{dB} , I''_{dB} , and M_{dB} provided previously to Equation 7 above, we obtain:

$$\text{Equation 8} \quad P\{SNR_{rcv} < SNR_{req}\} = P\{A_{dB} > M_{dB} - I''_{dB}\}$$

Although Equation 8 above is quite similar in appearance to Equation 1, the analysis and methodology provided in ITU-R P.530 to calculate the probability that the receive SNR is less than the required SNR is not directly applicable to our situation since both A_{dB} and I''_{dB} are r.v.'s. Thus, we must separately derive an expression to calculate the above probability.

We consider the r.v. I''_{dB} . Note that it takes on values i''_{dB} and that these value are discrete, ranging from 0 dB to an arbitrarily large value. For sufficiently small sample steps, we can approximate the right side of Equation 8 as follows:

$$\text{Equation 9} \quad P\{A_{dB} > M_{dB} - I''_{dB}\} \cong \sum_{i''_{dB}=0}^{\infty} [P\{A_{dB} > M_{dB} - i''_{dB}\} /_{I''_{dB}=i''_{dB}}] [P\{I''_{dB} = i''_{dB}\}] \Delta i''_{dB}$$

Observing Equation 9, we note that $p(I''_{dB} = i''_{dB})$ is in the form of a probability density function of I''_{dB} , (i.e., $f_{I''_{dB}}(i''_{dB})$). Additionally, for sufficiently small $\Delta i''_{dB}$, we can replace the summation in Equation 9 with an integral resulting in the following:

$$\begin{aligned} \text{Equation 10} \quad P\{A_{dB} > M_{dB} - I''_{dB}\} &\cong \int_{i''_{dB}=0}^{\infty} P\{A_{dB} > M_{dB} - i''_{dB}\} f_{I''_{dB}}(i''_{dB}) di''_{dB} \\ &= \int_{i''_{dB}=0}^{\infty} p_w(M_{dB} - i''_{dB}) f_{I''_{dB}}(i''_{dB}) di''_{dB} \end{aligned}$$

where the function p_w was defined previously in Equation 4.

We can see from Equation 10 that the integral includes the product of two terms. We observe that the first term has the same form as the expression calculated in ITU-R P.530, p_w , to determine the probability of the link fade, A_{dB} , exceeding some value, in this case $(M_{dB} - i''_{dB})$. Thus, the first term inside the integral is a function of $(M_{dB} - i''_{dB})$. The

second term is a function of i''_{dB} . The integral of the product of these two terms is exactly in the form for their convolution. That is, they are in the form below:

$$\text{Equation 11} \quad x(t) * y(t) \equiv \int_{-\infty}^{\infty} x(t - \tau) y(\tau) d\tau$$

where $*$ denotes convolution. Thus, we can re-write the right side of Equation 10 as follows:

$$\text{Equation 12} \quad \int_{i''_{dB}=0}^{\infty} p_w(M_{dB} - i''_{dB}) f_{I''_{dB}}(i''_{dB}) di''_{dB} = p_w * f_{I''_{dB}} \equiv p_f(M_{dB})$$

where we have defined a new function, p_f , that is dependent upon the link margin, M_{dB} , and represents the probability that the link fails.

Combining the results of Equation 12 above with Equation 8 we obtain the final expression:

$$\text{Equation 13} \quad P\{SNR_{rcv} < SNR_{req}\} = p_w * f_{I''_{dB}} \equiv p_f(M_{dB})$$

Thus, we can calculate the probability that the received SNR is less than the required SNR for a given link that is experiencing a random fade depth and a random interference signal level from a mobile satellite system by convolving the probability that the random fade exceeds the link margin, M_{dB} , with the probability density function of the r.v. I''_{dB} which represents the amount, in dB, that the received interference power exceeds the noise floor, N . We can generate the PDF of I''_{dB} via simulation using the MSS orbital and transmission characteristics and the FS receiver characteristics.

In some cases, it may not be straightforward to perform the convolution in Equation 13. For these cases, we can take advantage of the convolution theorem [Bracewell, 1978], which states that:

Equation 14
$$x(t)*y(t) = \mathfrak{S}^{-1}[\mathfrak{S}\{x(t)*y(t)\}] = \mathfrak{S}^{-1}[\mathfrak{S}\{x(t)\}\mathfrak{S}\{y(t)\}]$$

Where \mathfrak{S} and \mathfrak{S}^{-1} connote the Fourier transform and inverse Fourier transform, respectively. If we perform the Fourier transform discretely via the Fast Fourier Transform (FFT) technique, connote the inverse Fourier transform as IFFT, and apply the concepts in Equation 14, we can re-write Equation 13 as follows:

Equation 15
$$P\{SNR_{rcv} < SNR_{req}\} = IFFT[FFT\{P_w\}FFT\{f_{I''_{dB}}(i''_{dB})\}]$$

Equation 15 above can then be calculated for a range of i''_{dB} for any number of link margins, M_{dB} .

1.2 Probability Density Function Transformation

In order to apply the results of section 1.1, it is necessary to have probability density function (PDF) data for the random variable (r.v.) I''_{dB} which was defined earlier in section 1.1.2. I''_{dB} is an r.v. that represents the amount that the MSS interference power exceeds the noise floor, N, at the input to the FS receiver (expressed in dB). A common form of input data that can be generated by MSS operators is probability density function (PDF) data for the level of interference power, in dBW, arriving from the mobile satellite system (MSS) at the input to the fixed service (FS) receiver. In the previous section, we defined the MSS interference power as a r.v. referred to as I_{dBW} . Thus, a transformation will need to be performed on the PDF data for I_{dBW} in order to obtain the PDF data for I''_{dB} that is necessary in the above analysis. This section provides the derivation of the required transformation. Specifically, a procedure is described that can be used to generate PDF data for a random variable (r.v.) given a set of PDF data for a different but related r.v, which in our case are I_{dBW} and I''_{dB} , respectively. The primary reference used in this analysis is *Probability, Random Variables, and Stochastic Processes* [Papoulis, 1965].

1.2.1 General Transformation Equation

Assume there are two r.v.'s, X and Y, with the r.v. Y a function of X. To find the PDF of Y, $f_Y(y)$, for a given y we solve the equation:

Equation 16
$$y = g(x)$$

for x in terms of y . If x_1, x_2, \dots, x_n are all the real roots in solving the above equation, then the general form for transforming between the PDFs of X and Y is given by equation 5-6 of [Papoulis, 1965]:

Equation 17
$$f_Y(y) = \frac{f_X(x_1)}{|g'(x_1)|} + \frac{f_X(x_2)}{|g'(x_2)|} + \dots + \frac{f_X(x_n)}{|g'(x_n)|}$$

where

Equation 18
$$g'(x) = \frac{dg(x)}{dx}$$

1.2.2 Relationship Between I''_{dB} and I_{dBW}

The r.v.'s I and I'' were defined previously and an expression was given in Equation 6 describing the relationship between them. We now consider the relationship between I''_{dB} and I_{dBW} . Using Equation 6 we can write:

Equation 19
$$I''_{dB} = 10 \log \left(\frac{10^{\frac{I_{dBW}}{10}} + N}{N} \right)$$

1.2.3 Restatement of the Transformation Equation

Given the above relationship between I''_{dB} and I_{dBW} , we can develop an equivalent set of expressions to those provided in [Papoulis, 1965]. That is, we can state that:

Equation 20
$$\begin{aligned} X &\Rightarrow I_{dBW}; x \Rightarrow i_{dBW} \\ Y &\Rightarrow I''_{dB}; y \Rightarrow i''_{dB} \\ x_1, x_2, \dots, x_n &\Rightarrow i_{1,dBW}, i_{2,dBW}, \dots, i_{n,dBW} \end{aligned}$$

and

Equation 21
$$f_{I''_{dB}}(i''_{dB}) = \frac{f_{I_{dBW}}(i_{dBW} = i_{1,dBW})}{|g'(i_{1,dBW})|} + \frac{f_{I_{dBW}}(i_{dBW} = i_{2,dBW})}{|g'(i_{2,dBW})|} + \dots + \frac{f_{I_{dBW}}(i_{dBW} = i_{n,dBW})}{|g'(i_{n,dBW})|}$$

1.2.4 Application of the PDF Transformation

Noting relationship between I''_{dB} and I_{dBW} as defined by Equation 19 above (i.e., $I''_{dB} = g(I_{dBW})$), and applying Equation 16 and Equation 17, we must solve the following equation for i_{dBW} in terms of i''_{dB} :

Equation 22
$$i''_{dB} = g(i_{dBW}) = 10 \log \left(\frac{10^{\frac{i_{dBW}}{10}} + N}{N} \right)$$

The result is a single root for all i_{dBW} :

Equation 23
$$i_{dBW} = 10 \log \left(\frac{10^{\frac{i''_{dB}}{10}} - N}{N} \right) \Rightarrow i_{1,dBW}$$

Next, we calculate $g'(i_{1,dBW})$ which is simply $g'(I_{dBW} = i_{1,dBW})$. The following derivative relationships will be useful in calculating $g'(i_{1,dBW})$ and are from [Beyer, 1981]:

$$\frac{d(au)}{dx} = a \frac{du}{dx} \quad \text{with "a" a constant}$$

Equation 24
$$\frac{d(\log_a u)}{dx} = \frac{1}{\log_a e} \frac{1}{u} \frac{du}{dx}$$

$$\frac{d(a^u)}{dx} = a^u \log_e a \frac{du}{dx}$$

Applying the above relationships to Equation 22, we obtain:

Equation 25
$$g'(I_{dBW}) = 10 \log_{10} e \frac{N}{10^{\frac{I_{dBW}}{10}} + N} \frac{du}{dI_{dBW}}$$

where u is:

Equation 26
$$u = \frac{10^{\frac{I_{dBW}}{10}} + N}{N}$$

and du/dI_{dBW} is:

Equation 27
$$\frac{1}{N} \frac{1}{10^{\frac{I_{dBW}}{10}}} \log_e 10 \frac{1}{10}$$

Combining Equation 25 and Equation 27 and simplifying we obtain:

Equation 28

$$g'(I_{dBW}) = \frac{1}{1 + N \left(10^{\frac{-I_{dBW}}{10}} \right)}$$

Using Equation 23 for the value of $i_{1,dBW}$ and substituting into Equation 28 above, we calculate the following expression for $g'(I_{dBW} = i_{1,dBW})$ after some algebraic manipulation:

Equation 29

$$g'(I_{dBW} = i_{1,dBW}) = 1 - 10^{\frac{-i''_{dB}}{10}}$$

Finally, applying the results of Equation 23 and Equation 29 into Equation 21 above, we obtain the following transformation between the PDFs of the r.v.'s I_{dBW} and I''_{dB} :

$$\text{Equation 30} \quad f_{I''_{dB}}(i''_{dB}) = \frac{f_{I_{dBW}}(i_{dBW} = i_{1,dBW})}{|g'(i_{1,dBW})|} = \frac{f_{I_{dBW}}(i_{dBW} = 10 \log \left\{ N 10^{\frac{i''_{dB}}{10}} - N \right\})}{1 - 10^{\frac{-i''_{dB}}{10}}}$$

1.2.5 Example Calculation

Assume that we have PDF data for the r.v. I_{dBW} , the level of the MSS interference power, in dBW, present at the input to the FS receiver and we want to determine the PDF of I''_{dB} , the level, in dB, that the MSS interference power at the input to the FS receiver exceeds the noise floor, N, at a particular value, $i''_{dB} = 2$ dB. As seen in Equation 30, the transformation is a function of the noise floor value. Let us assume that the noise floor of the FS receiver is -140 dBW or 10^{-14} watts. Applying Equation 30, we calculate the following:

$$f_{I''_{dB}}(i''_{dB} = 2dB) = \frac{f_{I_{dBW}}(i_{dBW} = 10 \log \left\{ (10^{-14}) \left(10^{\frac{2}{10}} \right) - 10^{-14} \right\})}{-10^{\frac{-2}{10}}}$$

Equation 31

$$= (2.71) f_{I_{dBW}}(i_{dBW} = -142.36dBW)$$

Thus, the PDF of I''_{dB} evaluated at $i''_{dB} = 2dB$ is obtained by multiplying the PDF of I_{dBW} evaluated at $i_{dBW} = -142.36 dBW$ by the factor 2.71.

1.3 Recommended Approach for Specifying MSS/FS Interference Criteria

Using the methods derived in the previous sections, it is possible to calculate a reliability curve as a function of margin when an FS link is degraded by MSS interference. This curve can be compared to the noise only case in a number of ways and the results compared to MSS/FS interference criteria. This section describes several alternative methods for specifying MSS/FS interference criteria. (Note: In this section, the term “reliability” is used synonymously with “the probability of link failure,” understanding that either one is easily computed by subtracting the other from unity.)

1.3.1 Simple Reliability Limit

The first method presented for specifying MSS/FS interference criteria is called the "Simple Reliability Limit" and is illustrated in Figure 1. The figure illustrates both the noise-only and noise plus interference curves plotted as a function of link margin. The vertical axis is the logarithm of the probability of link failure by some criteria. As shown in the figure, the limit is in the form of a “corner.” If the curve representing reliability with noise and interference passes below and to the left of the corner, then reliability is considered acceptable. The location of the corner is determined horizontally by the actual existing margin of the link, X_0 . This can be computed knowing the received signal level (RSL) and the threshold level for the equipment. The vertical position of the corner is stated explicitly either as a reliability (e.g. 99.999%) or as the probability of link failure (e.g. 0.001%). Link reliability limits are discussed, for example, in TIA TSB-10-F, Section 4.2.

1.3.2 Reliability Degradation Limit

The second method presented for specifying MSS/FS interference criteria is called the "Reliability Degradation Limit" and is illustrated in Figure 2. In this method, the reliability is again considered at the actual link margin. However, the reliability limit is determined relative to the reliability of the link without interference. For example, a specification might state that the probability of link failure with interference must be no more than 25% greater than the probability of failure with noise alone. This approach is very similar to the fractional degradation in performance (FDP) described in ITU-R Recommendation IS.1141. Under certain conditions, the FDP can be computed based only on the average interference power.

1.3.3 Margin Degradation at Given Reliability

The third method presented for specifying MSS/FS interference criteria is called "Margin Degradation at Given Reliability" and is illustrated in Figure 3. In this case, a certain probability of link failure is given, and the specification requires that the margin be degraded no more than a certain number of decibels. In other words, the increase in signal level needed to maintain the given reliability must be no more than the specified number of dB. This method, like the reliability degradation limit method, requires knowledge of the reliability in the noise-only case, as well as the noise plus interference case.

1.3.4 Margin Degradation at Existing Margin

The final method presented for specifying MSS/FS interference criteria is called "Margin Degradation at Existing Margin" and is illustrated in Figure 4. This method is similar to the method specifying margin degradation at a given reliability. In this case, however, the reference is based on the actual margin, instead of a given reliability. In this fourth method, the increase in signal level needed to maintain the reliability of the system in the noise only case at the actual margin must be no more than the specified number of dB.

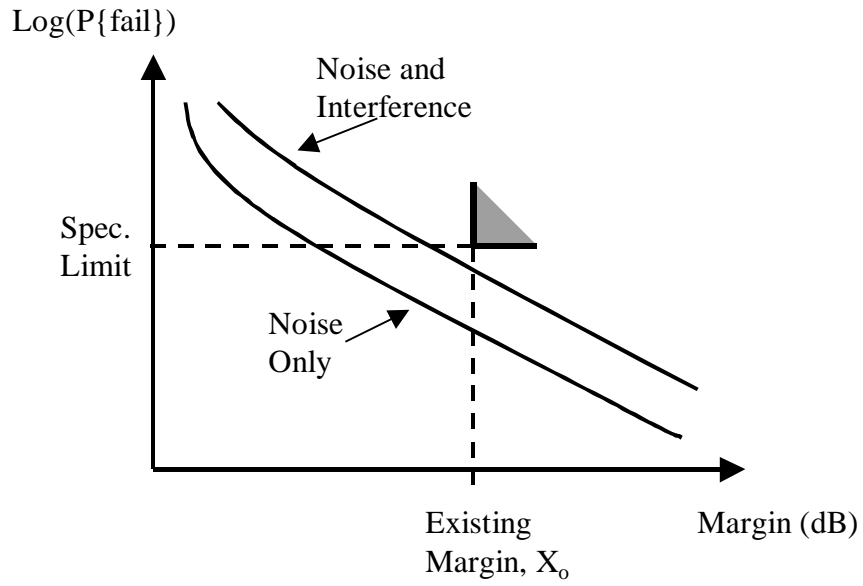


Figure 1: Simple Reliability Limit

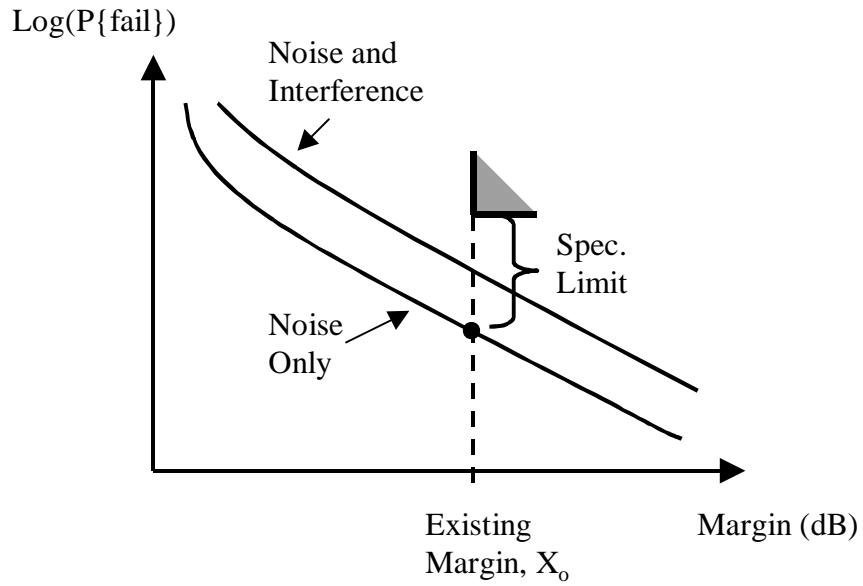


Figure 2: Reliability Degradation Limit

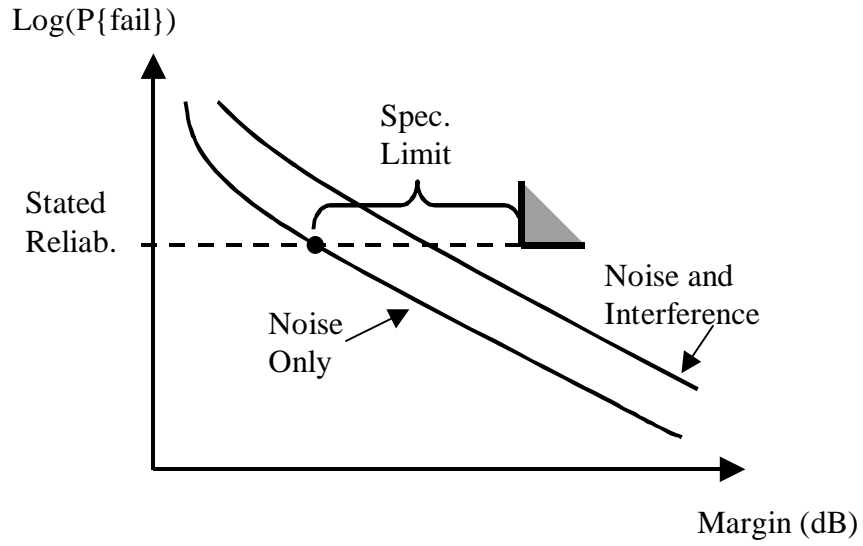


Figure 3: Margin Degradation Limit at Given Reliability

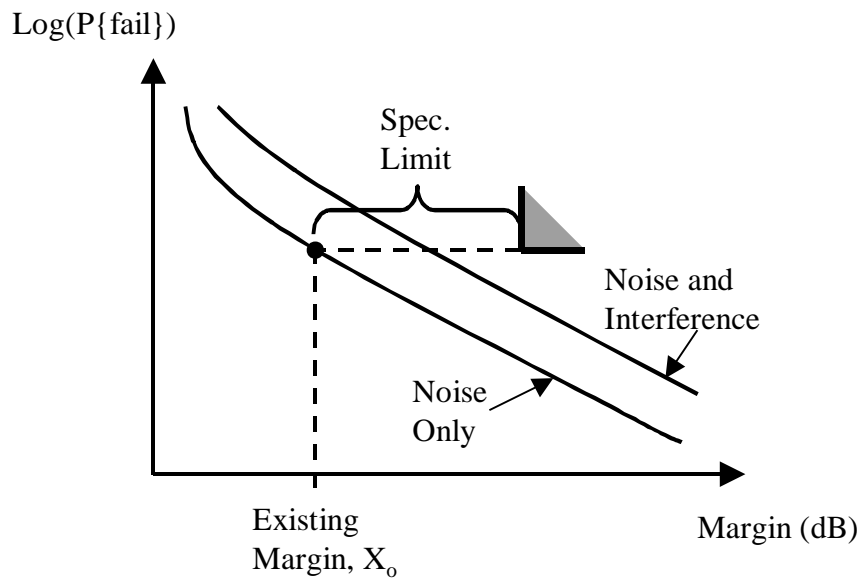


Figure 4: Margin Degradation Limit at Existing Limit

2 Summary and Conclusions

A methodology has been developed to quantify the impact of MSS space-to-Earth (S-E) transmission interference on FS radio-relay ground stations. This methodology is based on an approach that expands the existing probabilistic analysis in ITU-R P.530 by including the effect of MSS noise interference in the estimates of the probability of exceeding a particular fade margin on a radio path. Finally, several methods have been proposed for specifying MSS/FS interference criteria.

3 References

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