A Methodology for Calculating the Impact of MSS Space-to-Earth Interference on FS Radio-Relay Ground Stations

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Radio Dynamics

Setting the Stage

- The evaluation process should address all three aspects identified below

Input Data

What must be provided by FS and MSS Systems?

Analysis & Calculations

What to we do with the data?

Interference Criteria

How do we specify the forms and limits of interference?

Agenda

- Analysis and Calculations
  - Derivation
  - Application
- Input Data
- Interference Criteria
- Summary/Recommendations

This section will set a framework for discussing these sections

Derivation

- Goal: Calculate probability that a link fails for both the noise-only and noise plus MSS interference cases
  - In both cases, we need to quantify:

$$P\left[SNR_{rcv} < SNR_{req}\right]$$

  - The difference is that SNR_{rcv} changes for each case

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Case 1: Noise-Only

$$SNR_{rcv} = \frac{SNR_{link}}{A} = \frac{C/N}{A} = \frac{\left(P_t G_t G_r / N L_p \right)}{A}$$

Using the above expression, it can be shown that

$$P\left\{ SNR_{rcv} < SNR_{req} \right\} = P\left\{ A > \frac{SNR_{link}}{SNR_{req}} \right\}$$

Case 1: Noise-Only (Continued)

- If we define new variables that represent the receive and required SNR's in dB, we can derive the following expression:

$$P\left[SNR_{rcv,dB} < SNR_{req,dB} \right] = P\left(A_{dB} > M_{dB} \right)$$

$$\equiv p_w(M_{dB})$$

- Where p_w is the prob that the fade (in dB) exceeds the link (or fade) margin, M_{dB} (in dB)

Case 1: Noise-Only (Concluded)

- Note that p_w is precisely the variable defined by EQ 19 (large fades) and EQ 23 (small fades) of ITU-R P.530 vs7 when calculated in decimal form
- Question: Why not use TSB 10F instead of ITU-R P.530?
  - Addressed shortly

Case 2: Noise Plus MSS Interference

- SNR_{rcv} is now defined as follows

$$SNR_{rcv} = \frac{SNR_{link}}{A} = \frac{C/(N+I)}{A} = \frac{\left[P_t G_t G_r / (N+I) L_p \right]}{A}$$

where I is an r.v. that represents the level of MSS interference power, in watts, arriving at the FS receiver

Case 2: Noise Plus MSS Interference
(Continued)

- It now becomes convenient to define a new r.v., I”, that represents the amount that the MSS interference power exceeds the noise floor, N
  - Note: I” is a linear ratio (i.e., not in dB)

$$I'' = \frac{N + I}{N}$$

Case 2: Noise Plus MSS Interference
(Continued)

- If we substitute I" into the equation above for SNR_{rcv}, it can be shown that

$$P\left\{SNR_{rcv} < SNR_{req}\right\} = P\left\{\frac{P_t G_t G_r}{L_p A(N+I)} < SNR_{req}\right\}$$

$$\vdots$$

$$= P\left\{AI'' > \frac{SNR_{link}}{SNR_{req}}\right\}$$

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Case 2: Noise Plus MSS Interference
(Continued)

- If we define new r.v.'s that represent A and I'' in dB, we obtain

$$P\{SNR_{rcv} < SNR_{req}\} = P\{A_{dB} > M_{dB} - I''_{dB}\}$$

- The equation above is similar to the noise-only case but much more complex to calculate since both M_{dB} and I''_{dB} are r.v.'s

Case 2: Noise Plus MSS Interference
(Continued)

- We can approximate the right side of the above equation as follows

$$P\{A_{dB} > M_{dB} - I''_{dB}\} \cong$$

$$\sum_{i''_{dB}=0dB}^{\infty}\left[P\{A_{dB} > M_{dB} - I''_{dB}\} /_{I''_{dB}=i''_{dB}}\right]\left[P\{I''_{dB} = i''_{dB}\}\right]\Delta i''_{dB}$$

- Note that $P\{I''_{dB}=i''_{dB}\}$ is in the form of a PDF of I'' (i.e., $f_{I'',dB}(i''_{dB})$)

Case 2: Noise Plus MSS Interference
(Continued)

- Additionally, for sufficiently small $\Delta i''_{dB}$, we can replace the summation with an integral resulting in the following

$$P\left\{A_{dB} > M_{dB} - I''_{dB}\right\} \cong \int_{i''_{dB}=0}^{\infty} P\left\{A_{dB} > M_{dB} - i''_{dB}\right\} f_{I''_{dB}}(i''_{dB})\,di''_{dB}$$

$$= \int_{i''_{dB}=0}^{\infty} p_w(M_{dB} - i''_{dB})\, f_{I''_{dB}}(i''_{dB})\,di''_{dB}$$

Case 2: Noise Plus MSS Interference
(Continued)

- Observations
  - The first term has the same form as p_w defined in EQ 19 of ITU-R P.530 vs 7
    - P{fade exceeds some value M_{dB} - i''_{dB}}
    - It is a function of M_{dB} - i''_{dB}
  - The second term is a function of i''_{dB}
    - In fact, it is the PDF of i''_{dB}
  - The integral of the product of these two terms is exactly in the form of their convolution

Case 2: Noise Plus MSS Interference
(Continued)

- The convolution of two functions is defined by

$$x(t) * y(t) \equiv \int_{-\infty}^{\infty} x(t-\tau)\,y(\tau)\,d\tau$$

- By combining the preceding , we arrive at the final equation

$$P\{SNR_{rcv} < SNR_{req}\} = p_w * f_{I''_{dB}}$$

Case 2: Noise Plus MSS Interference
(Concluded)

- Thus, we can calculate the probability that the received SNR is less than the required SNR (i.e., the link fails) for a given link that is experiencing a random fade depth and a random MSS interference signal level by convolving p_w (the probability that the random fade exceeds the link margin, M_{dB}) with the probability density function of the r.v. I''_{dB}

Why Not Use TSB 10F Versus ITU-R P.530 to Calculate P{Link Fails}?

- EQ 4.2-2 of TSB 10F can be used to generate an expression that is similar to p_w in EQ 19 of ITU-R P.530 vs 7

$$T = \frac{r T_0 10^{-\left(CFM/10\right)}}{I_0}$$

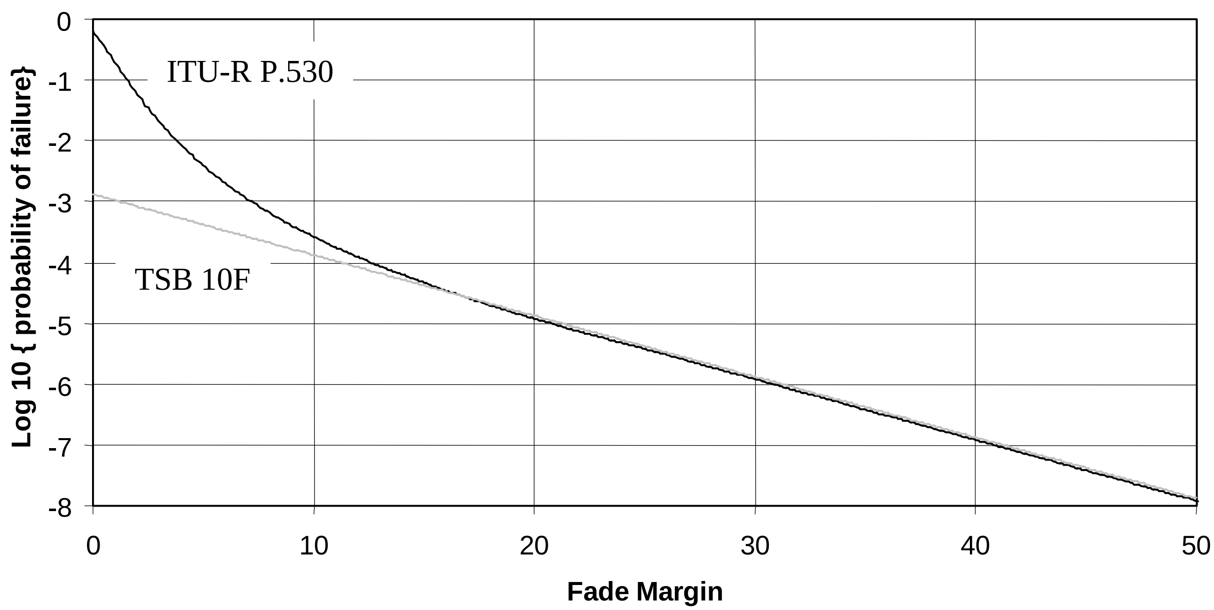
$$\vdots$$

$$p_{w_{TSB10F}} = 100\left(\frac{T}{T_0}\right) = 100(r)\left(10^{-\left(CFM/10\right)}\right)$$

* Assumes no diversity (I.e., $I_0 = 1$)

Why Not Use TSB 10F Versus ITU-R P.530 to Calculate P{Link Fails}? (Continued)

- However, TSB 10F equation focuses on large fades
  - Accuracy decreases rapidly for fades < ~15 dB



Antenna Size: 6 ft

Azimuth: 180 deg

Path Length: 10 km

Frequency: 2.18 GHz

$h_t = h_r = 400m$

Why Not Use TSB 10F Versus ITU-R P.530 to Calculate P{Link Fails}? (Concluded)

- "Small fade regime" is important when performing the convolution in the noise plus interference analysis
- Conclusion
  - Use ITU-R P.530 version of p_w versus EQ 4.2-2 of TSB 10F

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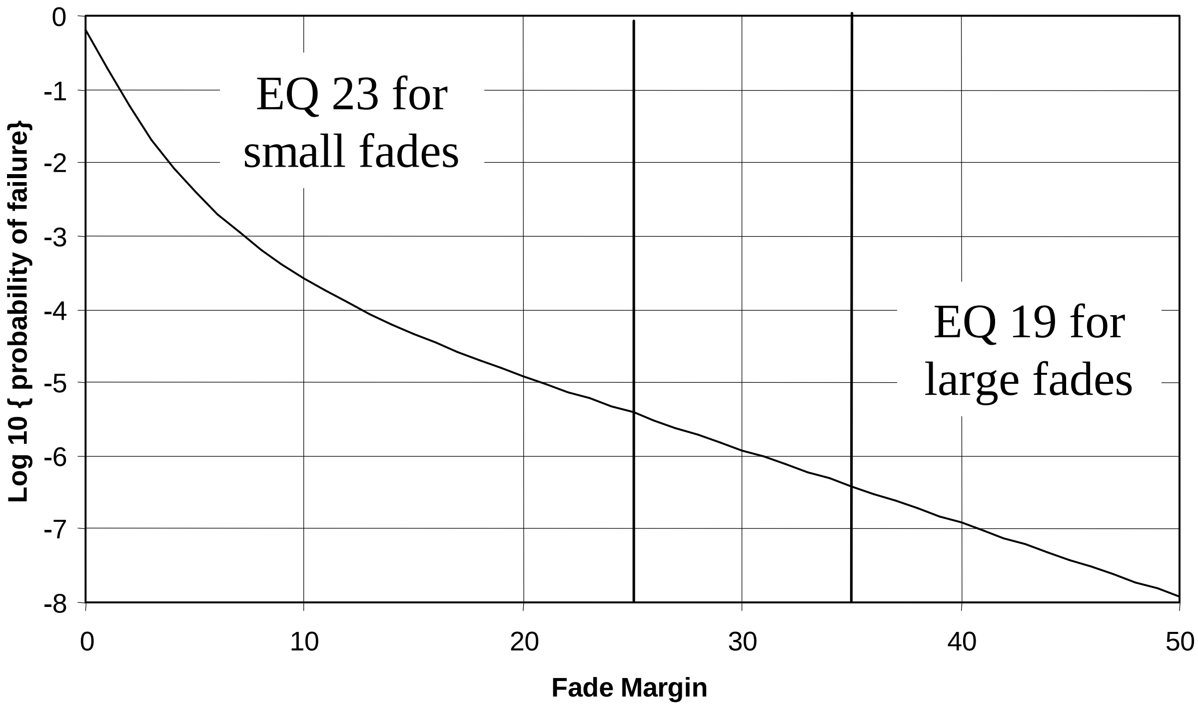
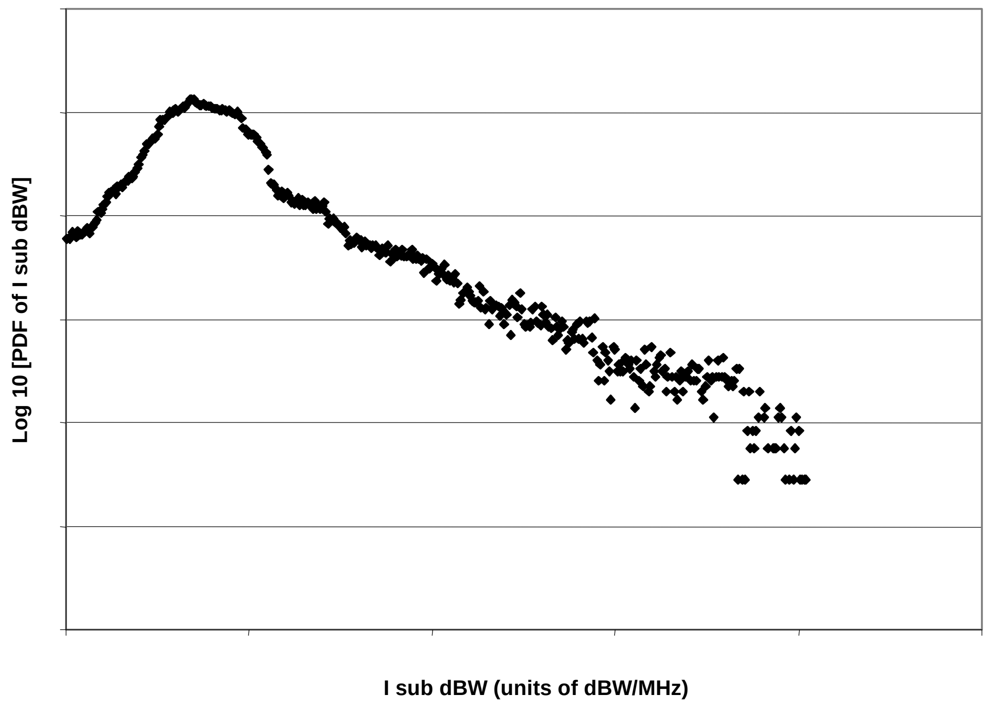
Application → Case 1: Noise-Only

- Step 1: Apply methodology of ITU-R P.530 vs 7



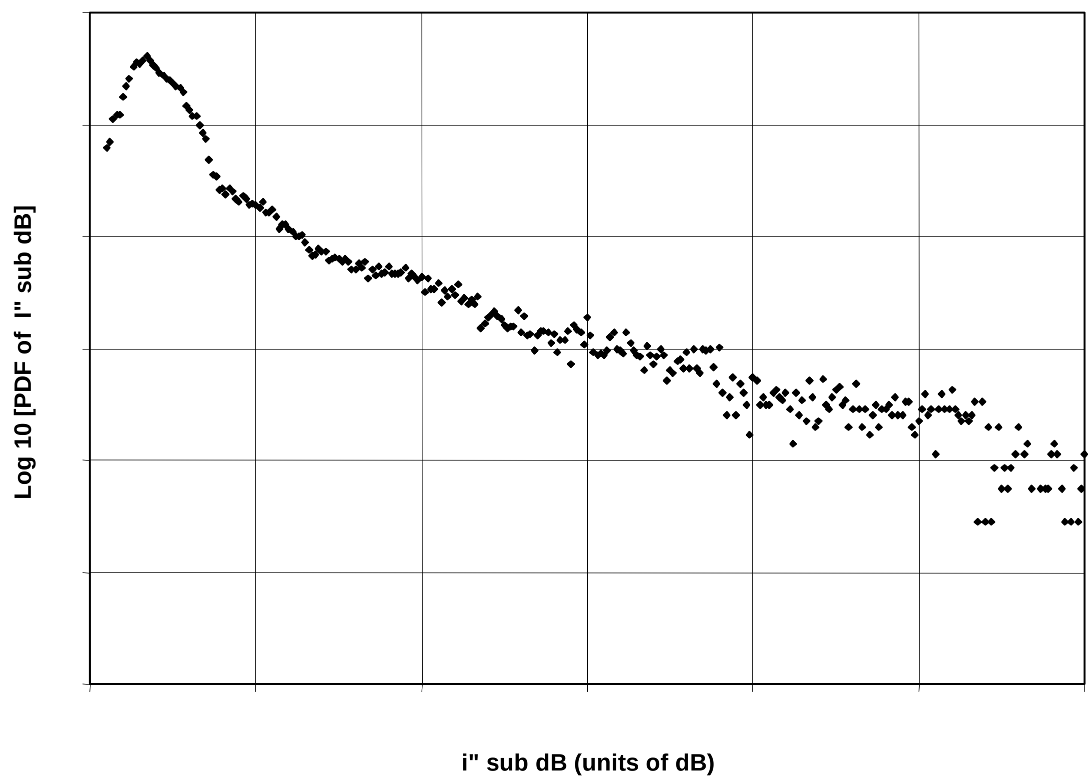
Application → Case 2: Noise Plus MSS Interference

- Step 1: Generate PDF data for MSS interference noise at FS receiver (i.e., for the r.v. I_{dBW})



Application → Case 2: Noise Plus MSS Interference (Continued)

- Step 2: Generate PDF data for the r.v. I''_{dB} from the data in step 1



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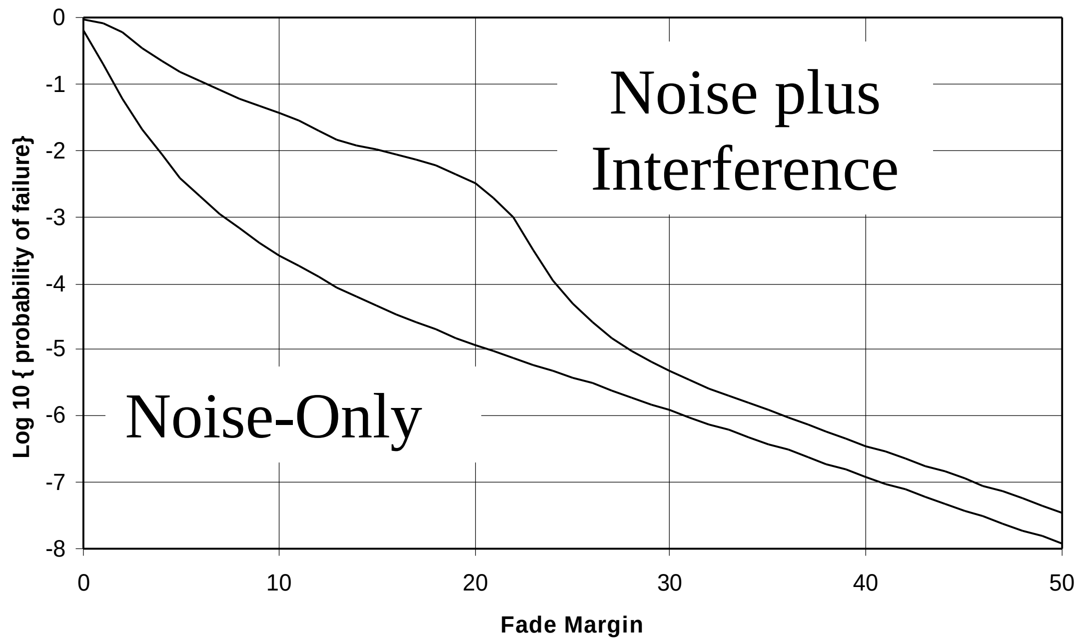
Application → Case 2: Noise Plus MSS Interference (Concluded)

- Step 3: Perform the convolution between p_w from ITU-R P.530 and the PDF data of I''_{dB} generated in step 2
  - Compare to "noise-only" case
  - A number of ways exist to interpret the results



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Observations

- Consideration must be given to how one would apply the proposed methodology to both analog and digital FS systems
  - Digital: Straightforward
  - Analog: Straightforward for “per hop” interference criteria limits
    - Will depend upon the “form” of the specified interference criteria
    - May require RF/Baseband transformation of the generated data or the specified criteria

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Input Data Considerations

FS System

- What is needed
  - Path design parameters
  - Example items include:
    - Antenna heights, gain and gain pattern characteristics
    - Receiver equipment characteristics
    - Terminal location
    - Path length
    - Frequency

MSS System

- What is needed
  - PDF of the MSS interference power at the FS receiver
- Several alternatives
  1. Directly provide PDF data of MSS interference power at FS RX vs. FS antenna and azimuth
  2. Provide PDF of power flux density (PFD) at FS antenna as a function of azimuth and elevation angle
  3. Provide orbital data plus PDF of satellite EIRP as a function of angle off boresight and satellite position

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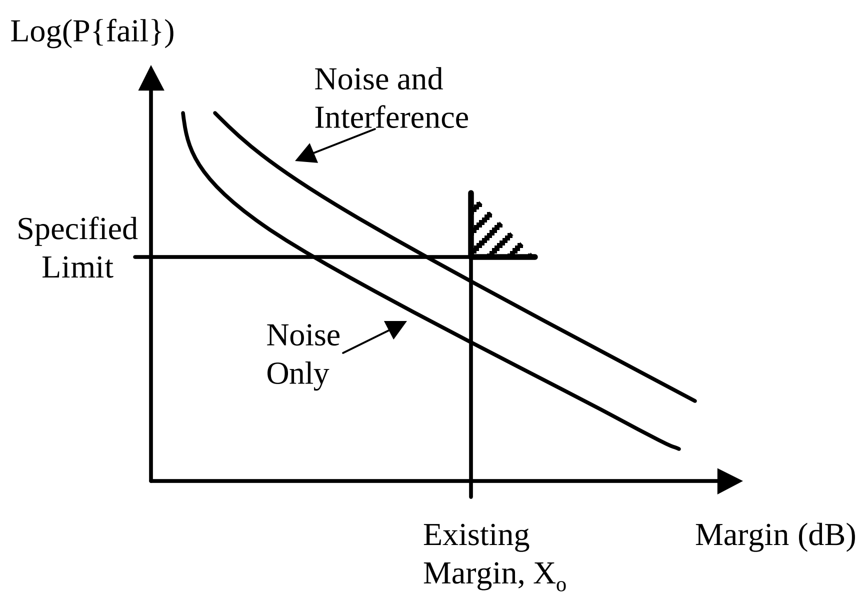
Alternative “Forms” for Specifying MSS/FS Interference Criteria

- Simple reliability limit
- Reliability degradation limit
- Margin degradation at given reliability
- Margin degradation at existing margin

Simple Reliability Limit



- Defined by a given reliability limit and an existing margin
  - Example: .99999 reliability at a margin of 35 dB
- Limit in form of "corner"
  - Require performance curve to be to the left of corner

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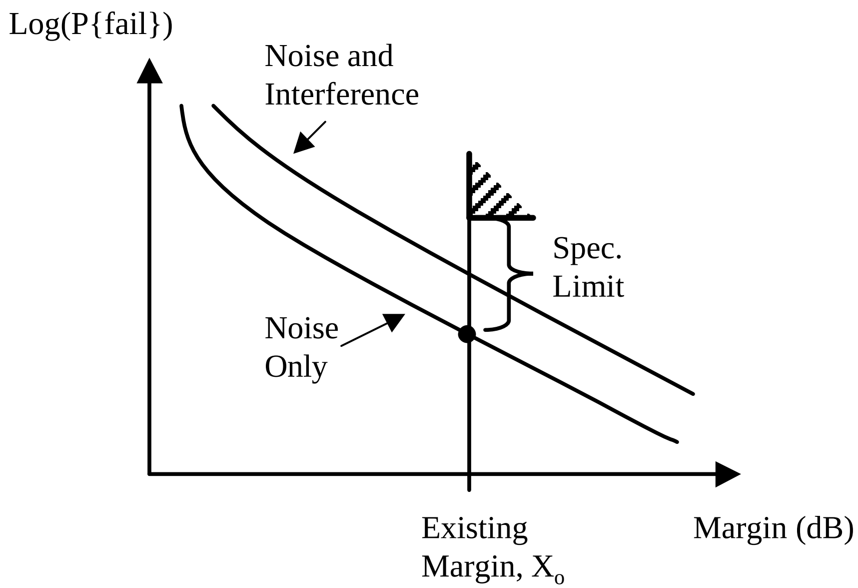
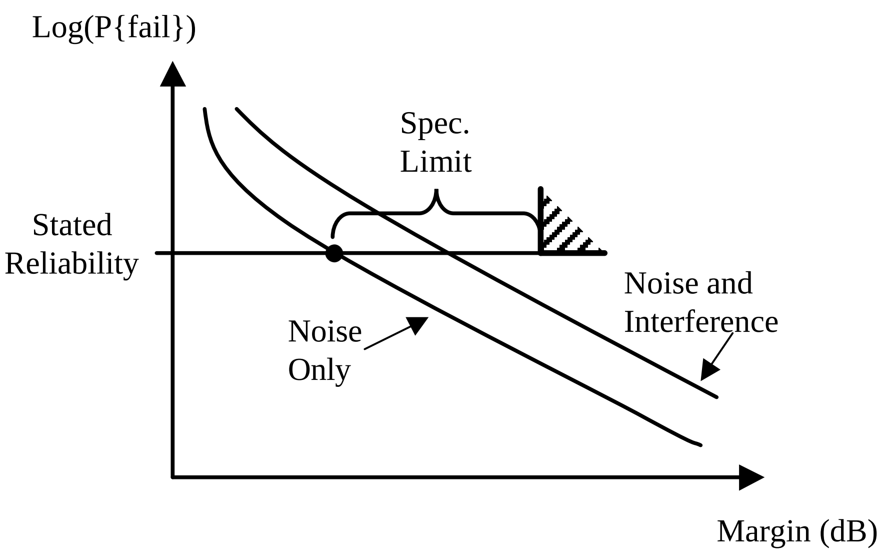
Reliability Degradation Limit



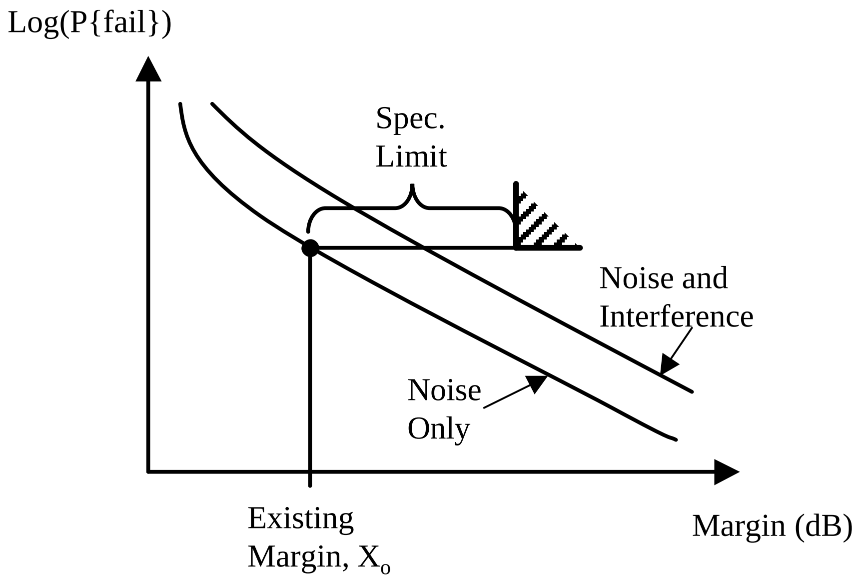
- Reliability limit determined at existing margin relative to link without interference
  - Example: No more than x% decrease in reliability at existing margin
- Similar to “FDP” approach in IS.1141

Margin Degradation at Given Reliability



- Given reliability point used as reference; limit is max degradation to margin allowed at given reliability due to MSS interference
  - Example: No more than x dB degradation to margin at .99999 reliability

Margin Degradation at Existing Margin



- Existing margin used as reference to determine reliability target; limit is max degradation to margin at reliability target due to MSS interference
  - Example: No more than x dB degradation to existing margin at existing reliability level

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Summary

- A methodology was developed to quantify MSS/FS interference
  - Builds upon results of ITU-R P.530 vs7
- Several alternatives were presented for:
  - Specifying the required input data for MSS systems
  - Specifying the limits of MSS/FS interference Criteria

Recommendations

- The JWG consider adopting the methodology presented to quantify MSS interference into FS receivers
- The JWG continue the dialogue regarding alternative “forms” for specifying:
  - MSS and FS input data
  - Interference criteria

Variable Definitions

C/N	=	Carrier to Noise power ratio (unitless)
N	=	Noise Power (watts)
P_t	=	Transmit Power (watts)
G_t	=	Transmit Gain (unitless)
G_r	=	Receive Gain (unitless)
L_p	=	Path Loss (unitless)
A	=	An r.v. that represents the link fade depth (expressed as a unitless factor)
M_{dB}	=	Link Margin (dB)

Variable Definitions (Continued)

A_{dB}	=	An r.v. that represents the link fade depth (expressed in dB)
$SNR_{rcv,dB}$	=	SNR_{rcv} in dB
$SNR_{req,dB}$	=	SNR_{req} in dB
$SNR_{link,dB}$	=	SNR_{link} in dB
K	=	Geoclimatic factor
d	=	Path length (km)
f	=	Frequency (GHz)
e_{p}, θ	=	Path Inclination (mrad, deg)

Variable Definitions (Continued)

T	=	Annual outage time (s)
r	=	Fade occurrence factor
T_0	=	Length of fade season (s)
CFM	=	Composite fade margin - digital (dB); fade margin - analog (dB)
I_0	=	Space diversity improvement factor
h_r	=	Rcv antenna height above sea level (m)
h_t	=	Xmit antenna height above sea level (m)

Variable Definitions (Concluded)

I = An r.v. that represents the level of interference power, in Watts, arriving from the mobile satellite system (MSS) at the input to the fixed service (FS) receiver

I_{dBW} = An r.v. that represents the level of interference power, in dBW, arriving from the mobile satellite system (MSS) at the input to the fixed service (FS) receiver

I'' = An r.v. that represents the amount that the MSS interference power exceeds the noise floor, N, at the input to the FS receiver (expressed as a dimensionless power ratio)

I''_{dB} = An r.v. that represents the amount that the MSS interference power exceeds the noise floor, N, at the input to the FS receiver (expressed in dB)